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EES351 2020/1 Part II.1 Dr.Prapun

3 Modulation and Communication Channels

3.1 Introduction to Modulation

Definition 3.1. The term **baseband** is used to designate the band of frequencies of the signal delivered by the source.

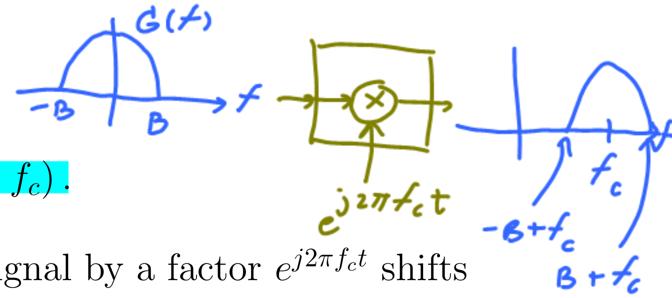
Example 3.2. In telephony, the baseband is the audio band (band of voice signals) of 0 to 3.5 kHz.

Definition 3.3. **Modulation**[11] is a process that causes a shift in the range of frequencies in a signal.

- Fundamental goal: produce an information-bearing modulated wave whose properties are best suited to the given communication task.
- The part of the system that performs this task is called the **modulator**.

Definition 3.4. In **baseband communication**, baseband signals are transmitted without modulation, that is, without any shift in the range of frequencies of the signal.

3.5. Recall the frequency-shift property:



$$e^{j2\pi f_c t} g(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} G(f - f_c).$$

This property states that multiplication of a signal by a factor $e^{j2\pi f_c t}$ shifts the spectrum of that signal by $\Delta f = f_c$.

[11] More general definition: modulation is the systematic alteration of one waveform, called the carrier, according to the characteristics of another waveform, the modulating signal or message. [3, p 162]

3.6. Frequency-shifting (frequency translation) is easily achieved by "multiplying" $g(t)$ by a sinusoid:

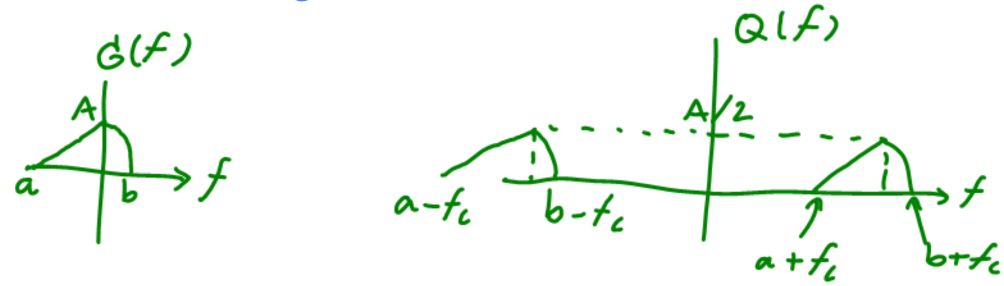
$$g(t) = g(t)\cos(2\pi f_c t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} \frac{1}{2}\left(G(f-f_c)+G(f+f_c)\right). \quad = Q(f) \tag{31}$$

$$\cos(\alpha) = \frac{1}{2}\left(e^{-j\alpha} + e^{j\alpha}\right)$$

$$\cos(2\pi f_c t) = \frac{1}{2}\left(e^{-j2\pi f_c t} + e^{j2\pi f_c t}\right)$$

$g(t)\times$

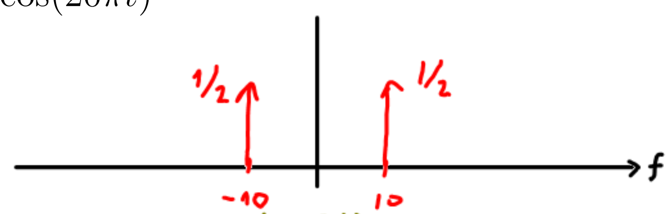
$$= \frac{1}{2} g(t) e^{j2\pi(-f_c)t} + \frac{1}{2} g(t) e^{j2\pi(f_c)t}$$



Example 3.7. Plot (by hand) the Fourier transforms of the following signals:

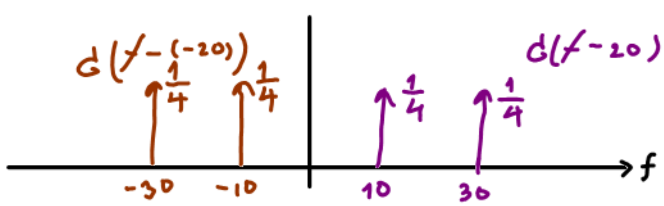
$2\pi(10)t$

(a) $\cos(20\pi t)$



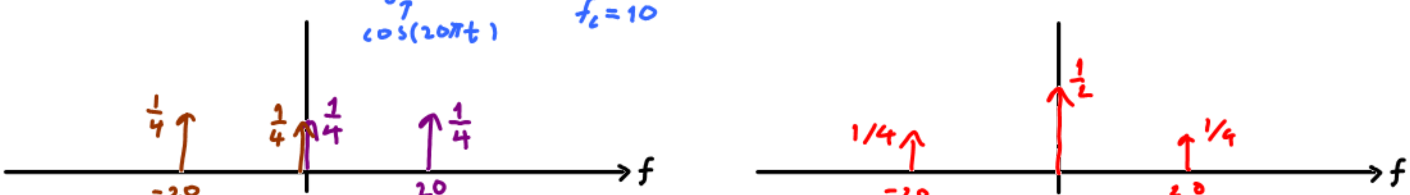
$g(t)$ $\quad$ $\cos(2\pi f_c t)$

(b) $\cos(20\pi t) \times \cos(40\pi t)$ $\quad f_c = 20$



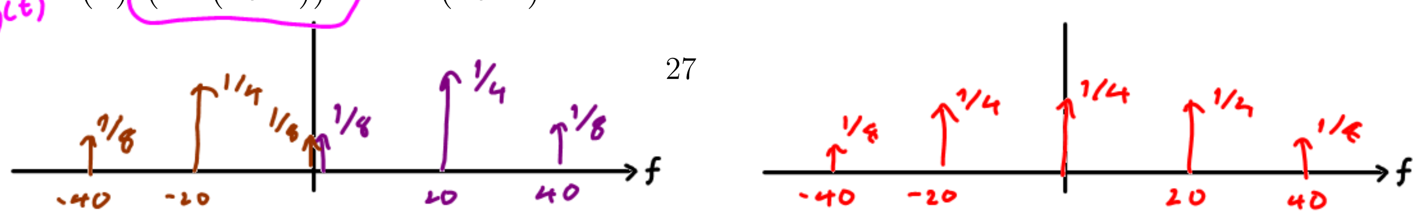
(c) $(\cos(20\pi t))^2 = g(t)\cos(20\pi t)$ $\quad f_c = 10$

$\cos(20\pi t)$



(d) $(\cos(20\pi t))^2 \times \cos(40\pi t)$

$g(t)$



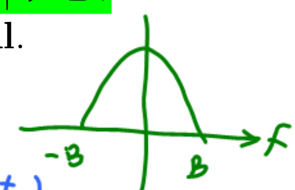
3.8. Similarly,

$$g(t)\cos(2\pi f_c t + \phi) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} \frac{1}{2}\left(G(f - f_c)e^{j\phi} + G(f + f_c)e^{-j\phi}\right). \tag{32}$$

Definition 3.9. The sinusoidal signals $\cos(2\pi f_c t)$ in (31) and $\cos(2\pi f_c t+\phi)$ in (32) are called the (sinusoidal) **carrier signals** and f_c is called the **carrier frequency**. In general, it can also has amplitude A and hence the general expression of the carrier signal is $A\cos(2\pi f_c t + \phi)$.

Definition 3.10. Communication that uses modulation to shift the frequency spectrum of a signal is known as **carrier communication**. [5, p 151]

Definition 3.11. We will use $m(t)$ to denote the baseband signal. We will assume that $m(t)$ is **band-limited** to B; that is, $|M(f)| = 0$ for $|f| > B$. Note that we usually call it the **message** or the **modulating signal**.



3.12. Time-domain effect of modulation:

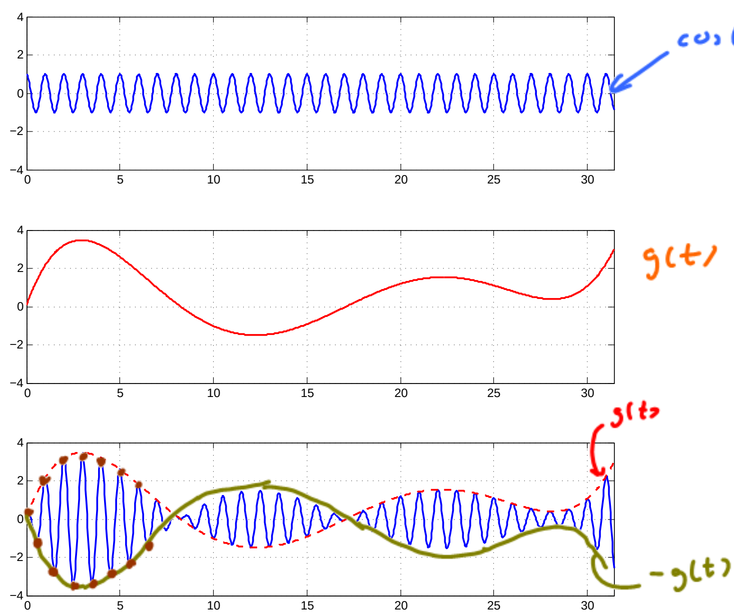


Figure 7: Simple Modulation: Time Domain

Example 3.13. A (Theoretically) Simple Modulator: Consider a **message** $m(t)$ produced by a source. Let the transmitted signal be

$$x(t) = \sqrt{2}\cos(2\pi f_c t) \times m(t).$$

Then,

$$X(f) = \frac{\sqrt{2}}{2} M(f - f_c) + \frac{\sqrt{2}}{2} M(f + f_c).$$

The block diagram for this modulator is shown in Figure 8 which also includes an example of the amplitude spectrum $|M(f)|$ for the message $m(t)$. With the given message spectrum and the carrier frequency being f_c, the amplitude spectrum $|X(f)|$ for the transmitted signal $x(t)$ is also shown.

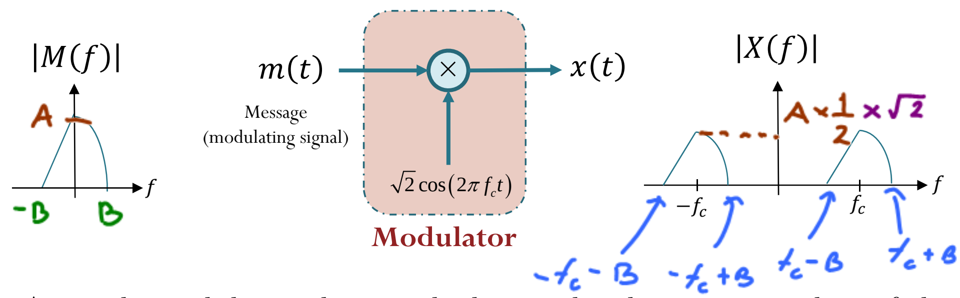


Figure 8: A simple modulator along with the amplitude spectrum plots of the signals.

Example 3.14. In Figure 9, an audio signal is used as the message $m(t)$.

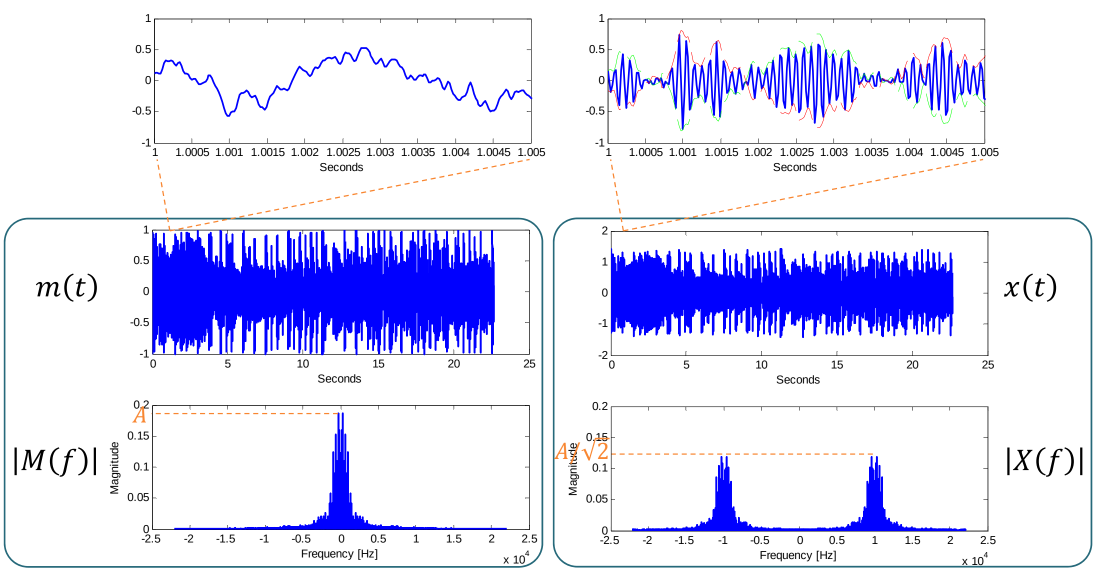


Figure 9: Signals in a simple modulator when an audio signal is used as its message.

Definition 3.15. The process of recovering the signal from the modulated signal (retranslating the spectrum to its original position) is referred to as **demodulation**, or **detection**.

3.16. Modulation (spectrum shifting) benefits and applications:

(a) **Reasonable antenna size**:

- For effective radiation of power over a radio link, the antenna size must be on the order of the wavelength[12] of the signal to be radiated.
- "Too low frequency" = "too large antenna"
- Audio signal frequencies are so low (wavelengths are so large) that impracticably large antennas will be required for radiation.
    - Shifting the spectrum to a higher frequency (a smaller wavelength) by modulation solves the problem.

$$\underset{3\times 10^8}{c} = \underset{3\text{ kHz}}{f}\lambda \Rightarrow \lambda = \frac{c}{f} = \frac{\cancel{3}\times 10^8}{\cancel{3}\times 10^3} = 10^5 = 100 \text{ km}$$

$$\underset{10^9}{3 \text{ GHz}} \Rightarrow \lambda = \frac{3\times 10^8}{3\times 10^9} = 0.1 \text{ m} = 10 \text{ cm}$$

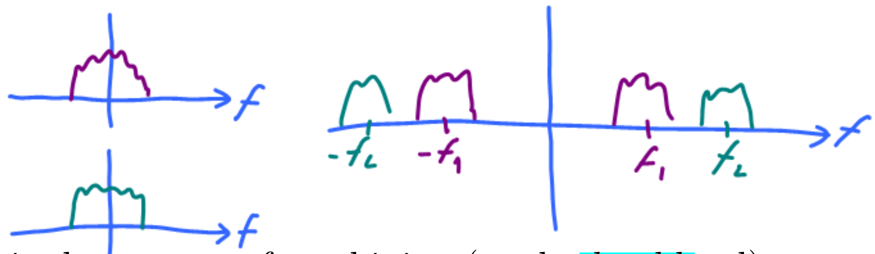
$$60 \text{ GHz} \Rightarrow \lambda = \frac{3\times 10^8}{60\times 10^9} = 5 \text{ mm}$$

(b) **Frequency Assignment**, **Frequency-Division Multiplexing** (FDM) and **Frequency-Division Multiple Access** (FDMA):

- If several signals (for example, all radio stations), each occupying the same frequency band, are transmitted simultaneously over the same transmission medium, they will all interfere.
    - Difficult to separate or retrieve them at a receiver.
    - One solution is to use modulation whereby each radio station is **assigned** a distinct carrier frequency.

[12] Efficient line-of-sight ratio propagation requires antennas whose physical dimensions are at least 1/10 of the signal's wavelength. [C&C [3], p. 8]

* Each station transmits a modulated signal, thus shifting the signal spectrum to its allocated band, which is not occupied by any other station.
* When you **tune** a radio or television set to a particular station, you are selecting one of the many signals being received at that time.
* Since each station has a different assigned carrier frequency, the desired signal can be separated from the others by filtering.



- **Multiplexing** is the process of combining (at the *local* level) several signals for simultaneous transmission on a common communications resource.
- **Multiple access** (MA) involves *remote* sharing of the resource.

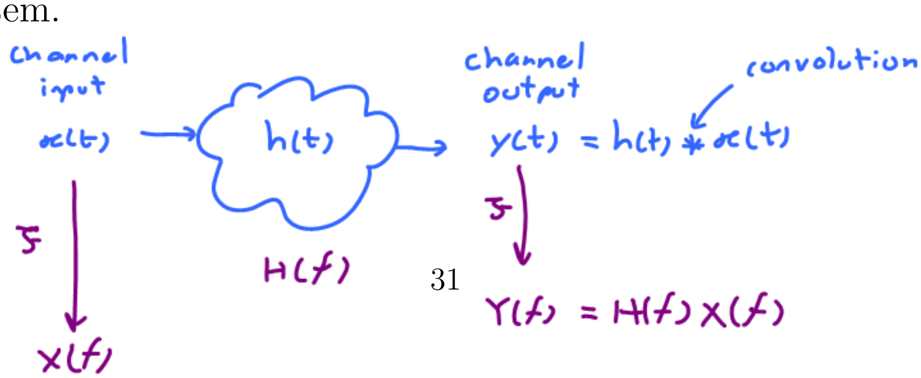
(c) **Channel passband matching**.

3.2 Communication Channel: Signal Distortion in Transmission

3.17. Recall that, for a linear, time-invariant (LTI) system, the input-output relationship is given by

$$y(t) = h(t) * x(t) \tag{33}$$

where $x(t)$ is the input, $y(t)$ is the output, and $h(t)$ is the **impulse response** of the system.



In which case,

$$Y(f) = H(f)X(f) \tag{34}$$

where $H(f)$ is called the **transfer function** or **frequency response** of the system. $|H(f)|$ and $\angle H(f)$ are called the **amplitude response** (or **gain**) and **phase response**, respectively. Their plots as functions of f show at a glance how the system modifies the amplitudes and phases of various sinusoidal inputs.

3.18. For a signal or an LTI system, because the time-domain representation and the frequency-domain representation are equivalent, in this class, when we draw a system diagram, we will not attempt to keep everything in the same domain. For example, you may see

$$X(f) \longrightarrow \boxed{h(t)} \longrightarrow y(t).$$

The input is given in its frequency-domain representation. The system and the output are given in their respective time-domain representations. It should be obvious from the variables "t" and "f" in an expression which domain is being used.

Note, however, that, to apply (33) or (34), all the functions that are plugged-in must be represented in the same domain.

3.19. From (34), observe that frequency response $H(f)$ is simply the weight that scales the the input $X(f)$ at each frequency f. Therefore, if the input contains only one frequency component (which is the case when we are dealing with a complex exponential function), the output is simply the input scaled by the value of $H(f)$ at that frequency:

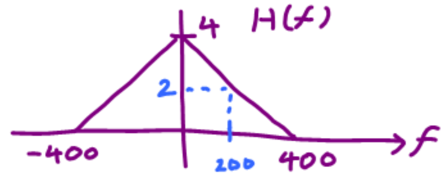
$$e^{j2\pi f_0 t} \longrightarrow \boxed{H(f)} \longrightarrow H(f_0)e^{j2\pi f_0 t}. \tag{35}$$

To see this, note that $X(f) = \delta(f - f_0)$ and by (34), we have $Y(f) = H(f)\delta(f - f_0) = H(f_0)\delta(f - f_0)$.

3.20. In general,

$$\sum_k a_k e^{j2\pi f_k t} \longrightarrow \boxed{H(f)} \longrightarrow \sum_k a_k H(f_k) e^{j2\pi f_k t}. \tag{36}$$

Example 3.21. Consider a communication channel whose frequency response is plotted below.



$$e^{j400\pi t} \longrightarrow \boxed{H(f)} \longrightarrow H(200)\,e^{j400\pi t} = 2e^{j400\pi t}$$

$$e^{j1000\pi t} \longrightarrow \boxed{H(f)} \longrightarrow H(500)\,e^{j1000\pi t} = 0\,e^{j1000\pi t} \equiv 0$$

$$\frac{1}{2}e^{j400\pi t} + \frac{1}{2}e^{-j400\pi t} = \cos(400\pi t) \longrightarrow \boxed{H(f)} \longrightarrow \frac{1}{2}\underbrace{H(200)}_{2}e^{j400\pi t} + \frac{1}{2}\underbrace{H(-200)}_{2}e^{-j400\pi t}$$

$$= 2\cos(400\pi t)$$

3.22. For a cosine input,

$$\cos(2\pi f_0 t) \longrightarrow \boxed{H(f)} \longrightarrow \frac{1}{2}H(f_0)e^{j2\pi f_0 t} + \frac{1}{2}H(-f_0)e^{j2\pi(-f_0)t}. \qquad (37)$$

Assume : $f_0 > 0$

$$= H(f_0)\left(\frac{1}{2}e^{j2\pi f_0 t} + \frac{1}{2}e^{-j2\pi f_0 t}\right)$$

$$= H(f_0)\cos(2\pi f_0 t)$$

If $H(-f_0) = H(f_0)$, then the output can be simplified to $H(f_0)\cos(2\pi f_0 t)$. Caution: From (35), it is tempting to conclude that when the input is $\cos(2\pi f_0 t)$, the corresponding output in (37) must be $H(f_0)\cos(2\pi f_0 t)$. This is not always the case. We need to check first that $H(-f_0) = H(f_0)$.

3.23. One can use (35) to estimate $H(f)$ by inputting a complex exponential test signal: Suppose we know that

$$ae^{j2\pi f_0 t} \longrightarrow \boxed{H(f)} \longrightarrow be^{j2\pi f_0 t}.$$

Then, we can infer that

$$H(f_0) = \frac{b}{a}.$$

Generalizing the result above, suppose we know that

$$\sum_k a_k e^{j2\pi f_k t} \longrightarrow \boxed{H(f)} \longrightarrow \sum_k b_k e^{j2\pi f_k t}.$$

Then, we can infer that

$$H(f_k) = \frac{b_k}{a_k}.$$

$a\left(\frac{1}{2}e^{j2\pi f_0 t}+\frac{1}{2}e^{-j2\pi f_0 t}\right)$ $b\left(\frac{1}{2}e^{j(2\pi f_0 t+\theta)}+\frac{1}{2}e^{-j(2\pi f_0 t+\theta)}\right)$

Example 3.24. Suppose we know that

$$a\cos(2\pi f_0 t) \longrightarrow \boxed{H(f)} \longrightarrow b\cos(2\pi f_0 t+\theta).$$

Then, we can infer that

$$H(f_0)=\frac{\frac{b}{2}e^{j\theta}}{\frac{a}{2}}=\frac{b}{a}e^{j\theta} \qquad H(-f_0)=\frac{\frac{b}{2}e^{-j\theta}}{\frac{a}{2}}=\frac{b}{a}e^{-j\theta}$$

Example 3.25. An "extremely nice" channel that does nothing to its input:

$$y(t)=x(t).$$

The output $y(t)$ when the input is $x(t)=\delta(t)$

Impulse response : $h(t)=\delta(t)$

$\downarrow \mathcal{F}$

Freq. response : $H(f)\equiv 1$

$H(f)=\frac{Y(f)}{X(f)}\equiv 1$

Definition 3.26. A channel is called **distortionless** if

$$y(t)=\beta x(t-\tau),$$

propagation delay $=\frac{\text{distance}}{c}$ (+processing delay)

real-valued

where β and τ are constants.

- The channel output has the same "shape" as its input.
- This is the "best" channel we can hope for. Any transmitted signal $x(t)$ will need to travel over some distance before it reaches the receiver. It will be delayed by the propagation time and its power will be attenuated.

Impulse Response : $h(t)=\beta\delta(t-\tau)$

Freq. response : $H(f)=\beta e^{-j2\pi f\tau}$

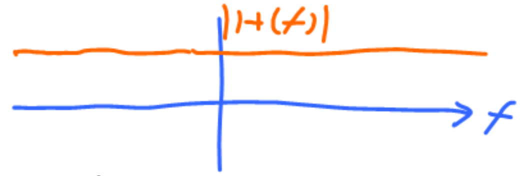
time-shift

$y(t)=\beta x(t-\tau)$

$\downarrow \mathcal{F}$

$Y(f)=\beta e^{-j2\pi f\tau}X(f)$

$H(f)=\frac{Y(f)}{X(f)}=\beta e^{-j2\pi f\tau}$



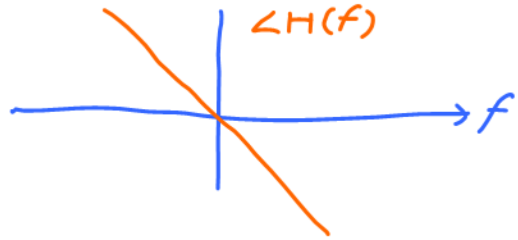
- A channel is distortionless if and only if it satisfies two properties:
  (a) "flat" frequency response: constant amplitude response

$$|H(f)| = |\beta e^{-j2\pi f\tau}| = |\beta||e^{-j2\pi f\tau}| = |\beta|$$

  (b) linear phase shift

$\beta > 0$

$$\angle H(f) = -2\pi\tau f$$



3.27. Major types of distortion

(a) Linear distortion

  (i) Amplitude distortion (frequency distortion): $H(f)$ is not constant with frequency.

  (ii) Phase distortion (delay distortion): the phase shift is not linear; the various frequency components suffer different amounts of time delay

(b) Nonlinear distortion: occur when the system includes nonlinear elements

Example 3.28. Amplitude distortion (frequency distortion)

(a) Figure 10 shows signals in a channel with low-frequency attenuation

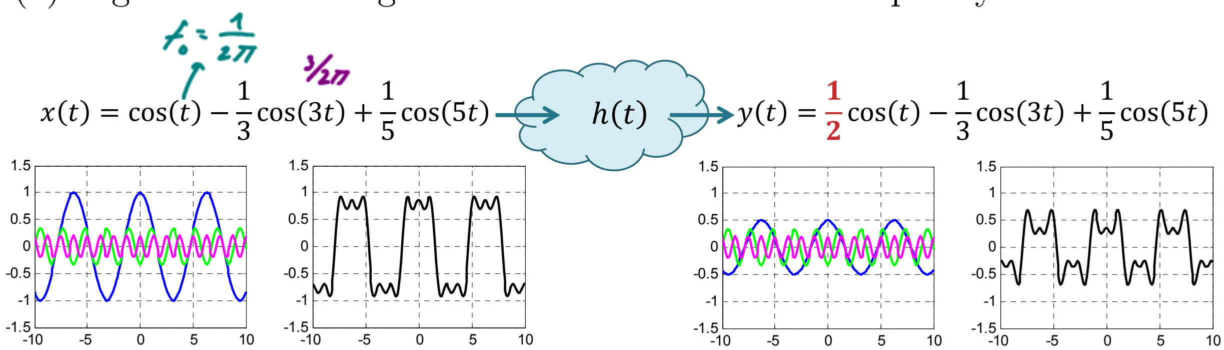


Figure 10: Channel with low-frequency attenuation

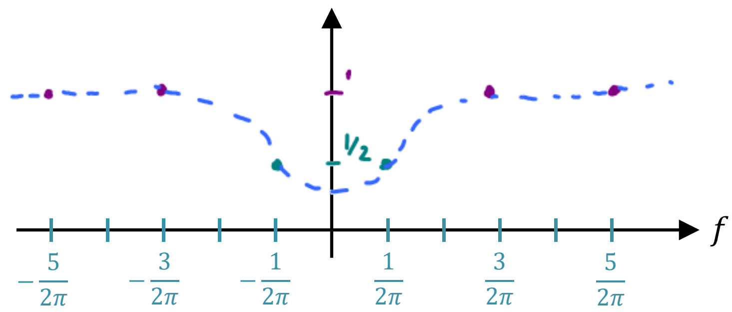


Figure 11: An example of $H(f)$ that satisfies the input-output relationship in Figure 10.

(b) Figure 12 shows signals in a channel with high-frequency attenuation

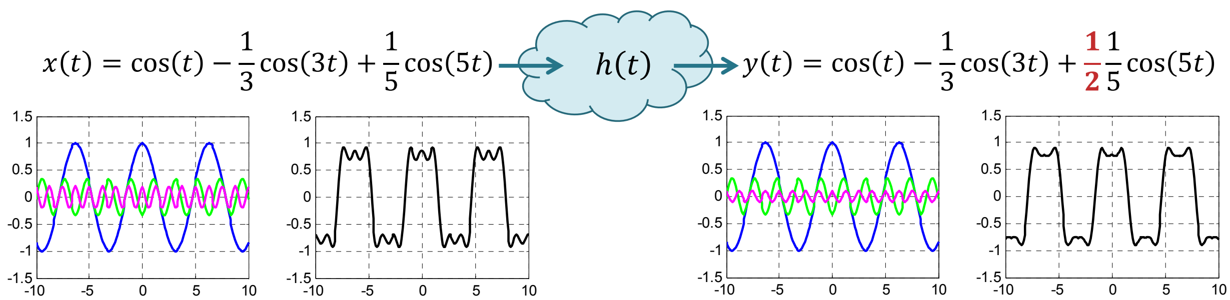


Figure 12: Channel with high-frequency attenuation

Example 3.29. A common area of confusion is **constant phase shift** versus **constant time delay**. The former, in general, causes distortion. The latter is desirable and is required for distortionless transmission.

(a) Figure 14 shows signals in a channel with constant phase shift

- Note that the peaks of the phase-shifted signal are substantially greater (by about 50 percent[13]) than those of the input test signal.
  - This is because the components of the distorted signal all attain maximum or minimum values at the same time, which was not true of the input.

[13]Delay distortion is crucial in pulse transmission. On the other hand, an untrained human ear is insensitive to delay distortion. Thus, delay distortion is seldom of concern in voice and music transmission.

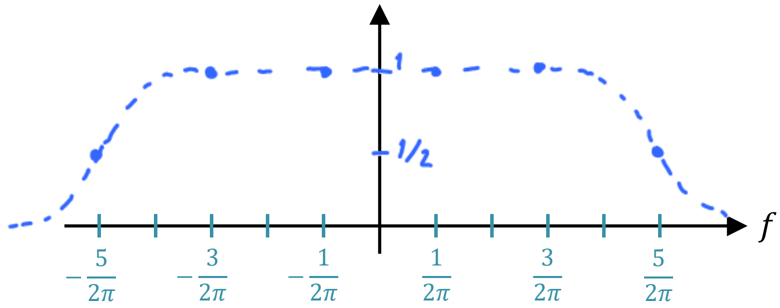


Figure 13: An example of $H(f)$ that satisfies the input-output relationship in Figure 12.

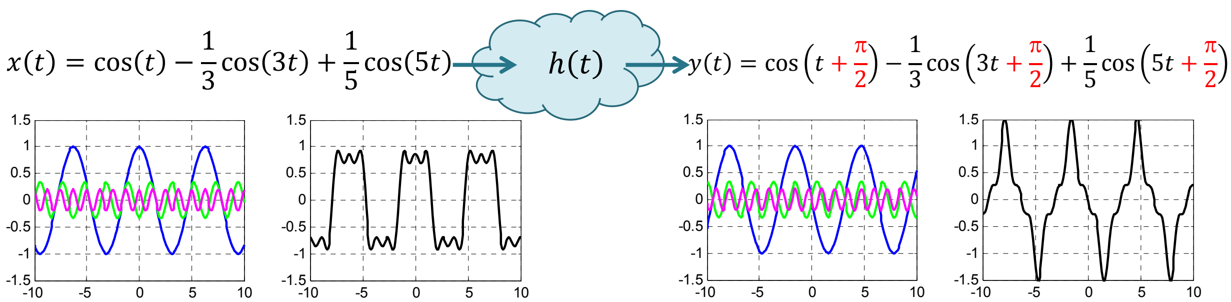


Figure 14: Channel with constant phase shift

$f_0 = \frac{1}{2\pi}, \frac{3}{2\pi}, \frac{5}{2\pi}$

$\theta = 90^\circ$

$H(f_0) = e^{j90^\circ} = j$

$H(-f_0) = e^{-j90^\circ} = -j$

- Frequency response:

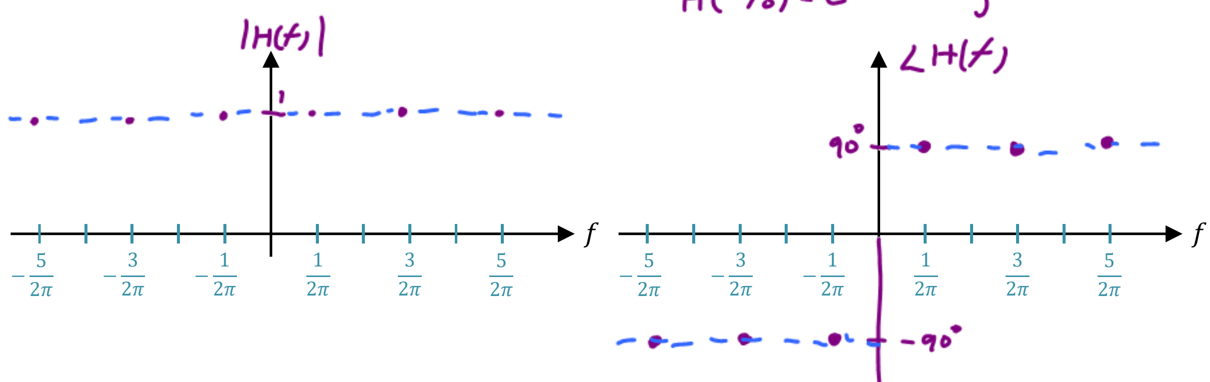


Figure 15: An example of $H(f)$ that satisfies the input-output relationship in Figure 14.

(b) Figure 16 shows signals in a channel with linear phase shift Note that

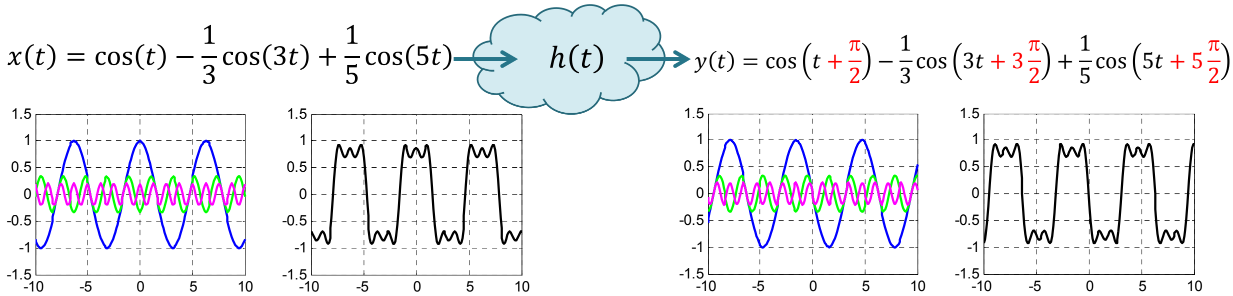


Figure 16: Channel with linear phase shift

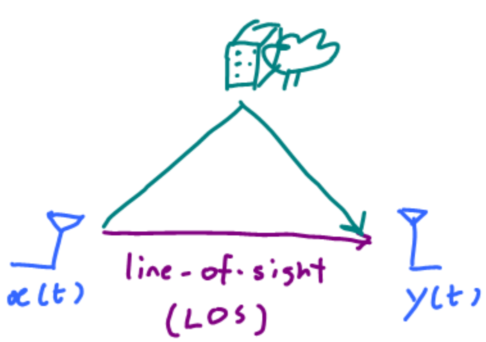
$$
\begin{aligned}
y(t) &= \cos\left(t + \frac{\pi}{2}\right) - \frac{1}{3}\cos\left(3t + 3\frac{\pi}{2}\right) + \frac{1}{5}\cos\left(5t + 5\frac{\pi}{2}\right) \\
&= \cos\left(t + \frac{\pi}{2}\right) - \frac{1}{3}\cos\left(3\left(t + \frac{\pi}{2}\right)\right) + \frac{1}{5}\cos\left(5\left(t + \frac{\pi}{2}\right)\right) \\
&= x\left(t + \frac{\pi}{2}\right)
\end{aligned}
$$

Therefore, this linear phase shift is the same as the time-shift operation.

3.30. Multipath Propagation and Time Dispersion [7, p 1]

- In *wireless* channel, the presence of multiple scatterers (buildings, vehicles, hills, and so on) causes the transmitted radio wave to propagate along several different paths (rays) that terminate at the receiver. Hence, the receive antenna picks up a superposition of *multiple attenuated and delayed copies* of the transmitted signal as shown in Figure 17.

  This phenomenon is referred to as **multipath propagation**. [7, p 1]



$y(t) = 0.9x(t - 5\text{ms}) + 0.5x(t - 6\text{ms})$

$\frac{\text{distance}}{c}$

- Due to different lengths of the propagation paths, the individual multipath components experience different delays (time shifts) [7, p 1]:

$$y(t) = x(t) * h(t) = \sum_{i=0}^{v} \beta_i x(t - \tau_i)$$

where

$$h(t) = \sum_{i=0}^{v} \beta_i \delta(t - \tau_i).$$

Here, $\beta_i = |\beta_i| e^{j\phi_i}$ and τ_i are, respectively, the complex attenuation factor and delay associated with the ith path.

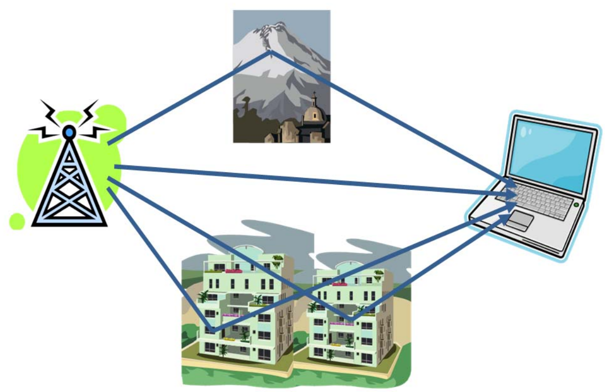


Figure 17: Multipath Propagation

- The receiver observes a temporally smeared-out version of the transmit signal. Such channels are termed **time-dispersive**
- The corresponding frequency response of the channel is

$$H(f) = \sum_{i=0}^{v} \beta_i e^{-j2\pi f \tau_i}. \tag{38}$$

- Time-dispersive channels are **frequency-selective** in the sense that different frequencies are attenuated differently. This is clear from the f-dependent $H(f)$ in (38).
    - These differences in attenuation become more severe when the difference of the path delays is large and the difference between the path attenuations is small. [7, p 3]

* This can be seen from the expression in Ex. 3.35.

- Although multipath propagation has traditionally been viewed as a transmission impairment, nowadays there is a tendency to consider it as beneficial since it provides additional degrees of freedom that are known as delay diversity or frequency diversity and that can be exploited to realize diversity gains or, in the context of multiantenna systems, even multiplexing gains. [7]

Example 3.31. Consider two propagation paths in a static environment. The receive signal is given by

$$y(t) = \beta_1 x(t-\tau_1) + \beta_2 x(t-\tau_2).$$

For this example, let's assume that the channel input $x(t) = \cos(2\pi f_c t)$. Furthermore, we also assume that β_1 and β_2 are real-valued.

$$y(t) = \beta_1 \cos(2\pi f_c(t-\tau_1)) + \beta_2 \cos(2\pi f_c(t-\tau_2))$$

$$= \beta_1 \cos(2\pi f_c t \underbrace{-2\pi f_c \tau_1}_{\theta_1}) + \beta_2 \cos(2\pi f_c t \underbrace{-2\pi f_c \tau_2}_{\theta_2})$$

phasor representation

$$\Leftrightarrow \beta_1 \angle \theta_1 + \beta_2 \angle \theta_2 \overset{\text{calculator}}{=} \beta \angle \theta$$

$$\Leftrightarrow \beta \cos(2\pi f_c t + \theta)$$

3.32. Consider sinusoids, all of which share the same frequency. Their linear combination is also a sinusoid at that shared frequency. The amplitude and phase of their combination can be found using phasors.

Example 3.33. $\cos(t) + \underset{\cos(t-90^\circ)}{\sin(t)} = \sqrt{2}\cos(t + (-45^\circ))$

$\cos(t) \Leftrightarrow 1\angle 0^\circ$, $\quad \sin(t) \Leftrightarrow 1\angle -90^\circ$

Example 3.34. $4\cos(2t) + 3\sin(2t)$

Example 3.35. Two-path channel: Consider two propagation paths in a static environment. The receive signal is given by

$$y(t) = \beta_1 x(t - \tau_1) + \beta_2 x(t - \tau_2).$$

In which case,

$$h(t) = \beta_1 \delta(t - \tau_1) + \beta_2 \delta(t - \tau_2)$$

and

$$H(f) = \beta_1 e^{-j2\pi f \tau_1} + \beta_2 e^{-j2\pi f \tau_2} = |\beta_1|\, e^{j\phi_1} e^{-j2\pi f \tau_1} + |\beta_2|\, e^{j\phi_2} e^{-j2\pi f \tau_2}.$$

Recall that $|Z_1 + Z_2|^2 = |Z_1|^2 + |Z_2|^2 + 2\mathrm{Re}\left\{Z_1 Z_2^*\right\}$. Therefore,

$$|H(f)|^2 = |\beta_1|^2 + |\beta_2|^2 + 2\,\mathrm{Re}\left\{|\beta_1|\, e^{j\phi_1} e^{-j2\pi f \tau_1} \left(|\beta_2|\, e^{j\phi_2} e^{-j2\pi f \tau_2}\right)^*\right\}$$

$$= |\beta_1|^2 + |\beta_2|^2 + 2\,|\beta_1|\,|\beta_2| \cos\left(2\pi\left(\tau_2 - \tau_1\right) f + \left(\phi_1 - \phi_2\right)\right).$$

Observation:

(a) In the frequency domain, the "frequency" of the oscillation is determined by $|\tau_2 - \tau_1|$.

(b) "Depth" of the fluctuation $= \frac{4|\beta_1||\beta_2|}{|\beta_1|^2 + |\beta_2|^2 + 2|\beta_1||\beta_2|}$

Example 3.36. Consider the two-path channels in which the receive signal is given by

$$y(t) = \beta_1 x(t - \tau_1) + \beta_2 x(t - \tau_2).$$

Four different cases are considered.

(a) Small $|\tau_1 - \tau_2|$ and $|\beta_1| \gg |\beta_2|$

(b) Large $|\tau_1 - \tau_2|$ and $|\beta_1| \gg |\beta_2|$

(c) Small $|\tau_1 - \tau_2|$ and $|\beta_1| \approx |\beta_2|$

(d) Large $|\tau_1 - \tau_2|$ and $|\beta_1| \approx |\beta_2|$

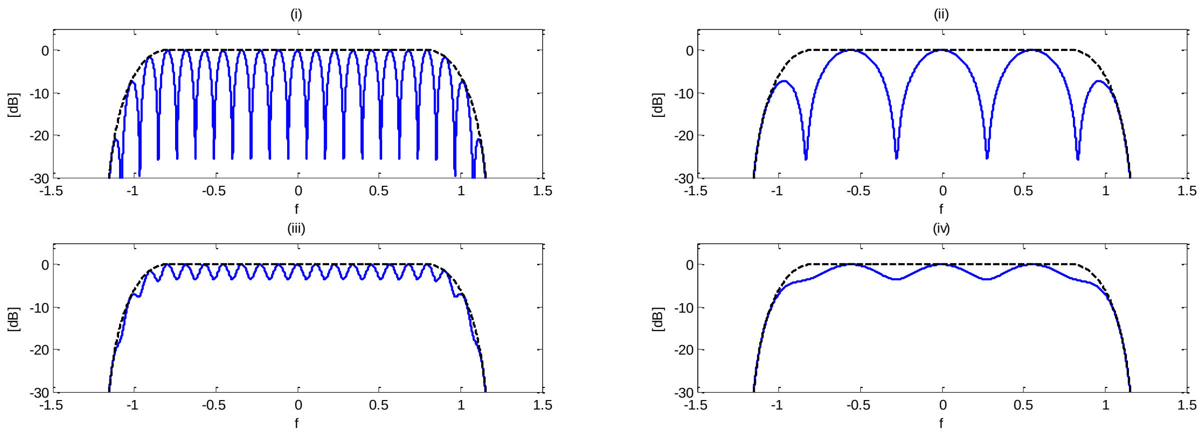


Figure 18: Frequency selectivity in the receive spectra (blue line) for two-path channels.

Figure 18 shows four plots of the normalized[14] $|X(f)|$ (dotted black line[15]) and the normalized $|Y(f)|$ (solid blue line) in [dB]. Match the four graphs (i-iv) to the four cases (a-d).

[14]The function is normalized so that the maximum point is 0 dB.

[15]For those who are curious, $x(t)$ is a raised cosine pulse with roll-off factor $\alpha = 0.2$ and symbol duration $T = 0.5$.